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Absorption of stopped K⁻ mesons by light nuclei

IL-TONG CHEON

Research Institute for Fundamental Physics, Kyoto University, Kyoto, Japan, and Kobe-Gakuin College, Kobe, Japan

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Abstract. The non-mesonic and non-radiative capture processes of K⁻ mesons in light nuclei, $K^- + 2N \rightarrow \Lambda + N$ and $K^- + 2N \rightarrow \Sigma + N$, have been investigated using a pair-absorption model. It is shown that the ratio of the capture rate by the relative D-state pair to that by the relative S-state pair is very sensitive to the short-range nuclear correlations.

1. Introduction

The capture of stopped K⁻ mesons by nuclei gives us information on some properties of light nuclei in the same way as π^- -meson absorption does (Cheon 1966, Eckstein 1963, Ericson 1962, Jibuti and Kopaleishvili 1964, Ozaki *et al.* 1960, Shklyarevskii 1964).

The capture process of K^- mesons is divided into two stages: (i) the atomic capture, (ii) the nuclear capture. The first stage provides a means of investigating the properties of the nuclear surface, while the second stage gives us some information on the correlations of the nucleons in the nucleus and on the mechanism of capture.

The K⁻-meson nuclear capture occurs through two processes:

- (I) mesonic process $K^- + N \rightarrow X + \pi$
- (II) non-mesonic process $K^- + 2N \rightarrow X + N$.

Most of the phenomena observed in K^- -meson capture by deuterium can be interpreted in terms of process (I) (Dahl *et al.* 1960, Wilkinson 1959, Fowler and Poulopoulos 1966). Processes of type (II) occur in heavier nuclei in a much larger proportion than process (I).

Most of the models for the basic reaction take little account of possible contributions from K⁻-meson absorption by light nuclei. Results obtained with the helium bubble chamber (Helium Bubble Chamber K⁻-Collaboration 1960) show, however, that a considerable fraction $(17 \pm 4\%)$ of the K⁻ mesons captured in helium do not produce π mesons. It is therefore reasonable to expect that similar reactions occur in the light nuclei, such as 1p-shell nuclei. Burhop *et al.* (1962) calculated the transition rate for nonmesonic capture processes of K⁻ mesons in a mesonic deuterium atom. Since the momentum transfer to the nucleons in such a process is large ($\simeq 911 \text{ Mev}/c$), only highly correlated pairs of nucleons with separation of the order of 0.4 fm seem to be able to exist. Therefore such a process must be useful as a means of studying nucleon positional correlations in the nucleus.

In the non-mesonic processes (II) the possible channels are as follows:

$$\begin{array}{ll} (1) \ \mathrm{K}^- + (\mathrm{pp}) \rightarrow \Lambda^0 + \mathrm{p} \\ (2) & \rightarrow \Sigma^0 + \mathrm{p} \\ (3) & \rightarrow \Sigma^+ + \mathrm{n} \\ (4) \ \mathrm{K}^- + (\mathrm{pn}) \rightarrow \Lambda^0 + \mathrm{n} \\ (5) & \rightarrow \Sigma^0 + \mathrm{n} \\ (6) & \rightarrow \Sigma^- + \mathrm{p} \\ (7) \ \mathrm{K}^- + (\mathrm{nn}) \rightarrow \Sigma^- + \mathrm{n}. \end{array}$$

In § 2 we calculate the ratio of the transition rate in the channel (4) to that in the channel (1). In § 3 the ratio of the K⁻ capture rate by the relative D-state pair to that by the relative S-state pair is obtained in the process (1). Furthermore, we discuss the effect of the nuclear correlations on this ratio. Finally, we calculate the absorption rate of the process (6). This process is easier to measure because only the charged particles are emitted.

2. Lambda particle emission processes

The interaction of the processes $K^- + 2N \rightarrow \Lambda + N$ is given in the form

$$\mathcal{H}^{\Lambda NK} = i g_{\Lambda NK} \bar{\psi}_{\Lambda} \gamma_5 \phi_K \psi_p \tag{1}$$

351

where $\bar{\psi}_{\Lambda}$ and ψ_{p} are the Λ -particle creation and proton annihilation operators. In the non-relativistic approximation the Hamiltonian can be rewritten as follows:

$$H^{\Lambda NK} = -G \sum_{i=1}^{2} \bar{\psi}_{\Lambda}(i) \{ \boldsymbol{\sigma}(i) \cdot \boldsymbol{\nabla}(i) \} \phi_{K}(i) \psi_{P}(i)$$
(2)

where

$$G = \frac{\hbar^2 g_{\Lambda \rm NK}}{2M_{\rm p} c (2m_{\rm K})^{1/2}} \frac{M_{\Lambda} - M_{\rm p}}{M_{\Lambda}}.$$

Under an LS coupling scheme the initial and final states are

$$|L_0 S^0 T\rangle_{\mathbf{I}} = \sum (L_c M_c \lambda \mu | L_0 M_0) (S \nu S' \nu' | S^0 M_S^0) (T' \Lambda' T_1 \Lambda_1 | T \Lambda) \Phi_c (L_c S' T')$$

$$\times \sum (lm L M | \lambda \mu) \langle nl N L; \lambda | n_1 l_1 n_2 l_2; \lambda \rangle V^{-1/2} \psi_{NLM} (\mathbf{R}) \varphi_{nlm} (\mathbf{r}) X_{S \nu \chi T_1 \Lambda_1}$$
(3)
$$L_c S' T' \rangle = V S' \delta (\mathbf{L} S' T') \langle nl N L | \mathbf{L} S \nu \lambda | \mathbf{L} S \nu \lambda \rangle V^{-1/2} \langle nl N L | \mathbf{L} S \nu \lambda \rangle V^{-1/2} \langle nl N L | \mathbf{L} S \nu \lambda \rangle V^{-1/2} \langle nl N L | \mathbf{L} S \nu \lambda \rangle V^{-1/2} \langle nl N L | \mathbf{L} S \nu \lambda \rangle V^{-1/2} \langle nl N L | \mathbf{L} S \nu \lambda \rangle V^{-1/2} \langle nl N L | \mathbf{L} S \nu \lambda \rangle V^{-1/2} \langle nl N L | \mathbf{L} S \nu \lambda \rangle V^{-1/2} \langle nl N L | \mathbf{L} S \nu \lambda \rangle V^{-1/2} \langle nl N L | \mathbf{L} S \nu \lambda \rangle V^{-1/2} \langle nl N L | \mathbf{L} S \nu \lambda \rangle V^{-1/2} \langle nl N L | \mathbf{L} S \nu \lambda \rangle V^{-1/2} \langle nl N L | \mathbf{L} S \nu \lambda \rangle V^{-1/2} \langle nl N L | \mathbf{L} S \nu \lambda \rangle V^{-1/2} \langle nl N L | \mathbf{L} S \nu \lambda \rangle V^{-1/2} \langle nl N L | \mathbf{L} S \nu \lambda \rangle V^{-1/2} \langle nl N L | \mathbf{L} S \nu \lambda \rangle V^{-1/2} \langle nl N L | \mathbf{L} S \nu \lambda \rangle V^{-1/2} \langle nl N L | \mathbf{L} S \nu \lambda \rangle V^{-1/2} \langle nl N L | \mathbf{L} S \nu \lambda \rangle V^{-1/2} \langle nl N L | \mathbf{L} S \nu \lambda \rangle V^{-1/2} \langle nl N L | \mathbf{L} S \nu \lambda \rangle V^{-1/2} \langle nl N L | \mathbf{L} S \nu \lambda \rangle V^{-1/2} \langle nl N L | \mathbf{L} S \nu \lambda \rangle V^{-1/2} \langle nl N L | \mathbf{L} S \nu \lambda \rangle V^{-1/2} \langle nl N L | \mathbf{L} S \nu \lambda \rangle V^{-1/2} \langle nl N L | \mathbf{L} S \nu \lambda \rangle V^{-1/2} \langle nl N L | \mathbf{L} S \nu \lambda \rangle V^{-1/2} \langle nl N L | \mathbf{L} S \nu \lambda \rangle V^{-1/2} \langle nl N L | \mathbf{L} S \nu \lambda \rangle V^{-1/2} \langle nl N L | \mathbf{L} S \nu \lambda \rangle V^{-1/2} \langle nl N L | \mathbf{L} S \nu \lambda \rangle V^{-1/2} \langle nl N L | \mathbf{L} S \nu \lambda \rangle V^{-1/2} \langle nl N L | \mathbf{L} S \nu \lambda \rangle V^{-1/2} \langle nl N L | \mathbf{L} S \nu \lambda \rangle V^{-1/2} \langle nl N L | \mathbf{L} S \nu \lambda \rangle V^{-1/2} \langle nl N L | \mathbf{L} S \nu \rangle V^{-1/2} \langle nl N L | \mathbf{L} S \nu \rangle V^{-1/2} \langle nl N L | \mathbf{L} S \nu \rangle V^{-1/2} \langle nl N L | \mathbf{L} S \nu \rangle V^{-1/2} \langle nl N L | \mathbf{L} S \nu \rangle V^{-1/2} \langle nl N L | \mathbf{L} S \nu \rangle V^{-1/2} \langle nl N L | \mathbf{L} S \nu \rangle V^{-1/2} \langle nl N L | \mathbf{L} S \nu \rangle V^{-1/2} \langle nl N L | \mathbf{L} S \nu \rangle V^{-1/2} \langle nl N L | \mathbf{L} S \nu \rangle V^{-1/2} \langle nl N L | \mathbf{L} S \nu \rangle V^{-1/2} \langle nl N L | \mathbf{L} S \nu \rangle V^{-1/2} \langle nl N L | \mathbf{L} S \nu \rangle V^{-1/2} \langle nl N L | \mathbf{L} S \nu \rangle V^{-1/2} \langle nl N L | \mathbf{L} S \nu \rangle V^{-1/2} \langle nl N L | \mathbf{L} S \nu \rangle V^{-1/2} \langle nl N L | \mathbf{L} S \nu \rangle \rangle V^{-1/2} \langle nl N L | \mathbf{L} S \nu \rangle \rangle V^{-1/2} \langle nl N L | \mathbf{L} S$$

$$|L_{c}S'T'\rangle_{F} = V^{-3/2} \exp\{i(\mathbf{K}_{c} \cdot \mathbf{X} + \mathbf{k}_{1} \cdot \mathbf{r}_{1} + \mathbf{k}_{2} \cdot \mathbf{r}_{2})\}\Phi_{c}(L_{c}S'T')X_{S''\nu''}$$
(4)

where $\Phi_c(L_cS'T')$ denotes the state of the core and/or residual nucleus and $\psi_{NLM}(\mathbf{R})$ and $\varphi_{nlm}(\mathbf{r})$ represent the wave functions of the centre-of-mass and the relative motions of the two emitted particles. We can rewrite the final state with the coordinates of the centre-of-mass and relative motions of the two emitted particles:

$$L_{\rm o}S'T'\rangle_{\rm F} = V^{-3/2}\exp\{i(\mathbf{K}_{\rm o}+\mathbf{K})\cdot\mathbf{R}_{\rm o}+i(\boldsymbol{\xi}\mathbf{K}-\boldsymbol{\eta}\mathbf{K}_{\rm o})\cdot\mathbf{R}+i\mathbf{k}\cdot\mathbf{r}\}\Phi_{\rm o}(L_{\rm o}S'T')X_{S''v''}$$
(5)

where

$$\begin{split} \boldsymbol{\xi} &= \frac{M_{\rm c}}{M_{\rm c} + M_{\Lambda} + M_{\rm p}}, \qquad \eta = \frac{M_{\Lambda} + M_{\rm p}}{M_{\rm c} + M_{\Lambda} + M_{\rm p}} \\ \mathbf{k} &= \mathbf{k}_1 + \mathbf{k}_2, \qquad \mathbf{k} = \frac{M_{\rm p} \mathbf{k}_1 - M_{\Lambda} \mathbf{k}_2}{M_{\Lambda} + M_{\rm p}}. \end{split}$$

In this scheme the absorption rate is given in the following form:

$$W = \frac{2\pi}{\hbar} \int \sum_{av} |\langle L_{c}S'T'|H^{\Lambda NK}|L_{0}S^{0}T\rangle|^{2} \frac{V^{3}}{(2\pi)^{9}} \frac{1}{2} \left(\frac{2\mu}{\hbar^{2}}\right)^{3/2} \\ \times \left\{m_{K}c^{2} - (M_{\Lambda} - M_{p})c^{2} - B - E_{x} - \frac{\hbar^{2}K^{2}}{2(M_{\Lambda} + M_{p})} - \frac{\hbar^{2}K_{c}^{2}}{2M_{c}}\right\}^{1/2} d\mathbf{K} \, d\mathbf{K}_{c} \, d\Omega_{K}$$
(6)

when $m_{\rm K}$, M_{Λ} , $M_{\rm p}$ and $M_{\rm c}$ are the masses of K⁻, Λ , proton and residual nucleus; B and $E_{\rm x}$ are the separation and excitation energies, respectively, and $1/\mu = 1/M_{\Lambda} + 1/M_{\rm p}$.



Figure 1. The scheme of the final state.

In the spin and isobaric-spin space the matrix elements are given such that

$$L^{\text{ANK}}(^{1}\text{pp}) = 2G\delta_{S''}(Q^{+}\delta_{\nu''-1} - Q^{0}\delta_{\nu''} + Q^{-}\delta_{\nu''-1})$$

for K^- absorption by the pp pair,

$$L^{\Lambda NK}(^{1}pn) = 2G\delta_{S''1}(P^{+}\delta_{\nu''-1} - P^{0}\delta_{\nu''0} + P^{-}\delta_{\nu''1})$$

for the singlet pn pair and

$$L^{\Lambda NK}(^{3}pn) = 2G\delta_{S''1}\{Q^{+}(\delta_{\nu''0}\delta_{\nu 1} + \delta_{\nu''1}\delta_{\nu 0}) + Q^{0}(\delta_{\nu''-1}\delta_{\nu-1} - \delta_{\nu''1}\delta_{\nu 1}) - Q^{-}(\delta_{\nu''1}\delta_{\nu 0} + \delta_{\nu''0}\delta_{\nu-1})\} + 2G\delta_{S''0}\delta_{\nu''0}(P^{+}\delta_{\nu 1} + P^{0}\delta_{\nu 0} + P^{-}\delta_{\nu-1})$$

for the triplet pn pair, where we define

$$\begin{split} P &= \Phi \bigtriangledown_{R} + \phi \bigtriangledown_{r} \\ Q &= 2\Phi \bigtriangledown_{r} + \frac{1}{2}\phi \bigtriangledown_{R} \\ \Phi &= \frac{1}{2} \{ \phi_{\mathrm{K}}(1) + \phi_{\mathrm{K}}(2) \}, \qquad \phi = \phi_{\mathrm{K}}(1) - \phi_{\mathrm{K}}(2). \end{split}$$

Furthermore, the operators with superscripts are defined by

$$\nabla^{\pm} = \mp \frac{1}{\sqrt{2}} (\nabla_x \pm i \nabla_y), \qquad \nabla^0 = \nabla_z.$$

The wave function for the K⁻ meson bound to a light nucleus can be explicitly expressed by Laguerre polynomials:

$$\phi_{n'l'm'}(\mathbf{x}) = -\left[\frac{(n'-l'-1)!}{2n'\{(n'+l')!\}^3}\right]^{1/2} \left(\frac{2Z}{n'a_0}\right)^{l'+\frac{3}{2}} \\ \times x^{l'} \exp\left(-\frac{Z}{n'a_0}x\right) L_{n'+l'}^{2l'+1} \left(\frac{2Z}{n'a_0}x\right) Y_{l'm'}(\hat{x})$$
(7)

where Z is the atomic number of the initial nucleus and a_0 is the Bohr radius of the K-mesonic atom. Since the bound-state wave function of K⁻ mesons scarcely varies in the region of the nucleus, an approximation $\phi_{\rm K}(1) \simeq \phi_{\rm K}(2) \simeq \phi_{\rm K}({\bf R})$ is allowed. Thus the operators P and Q become

$$P = \phi_{\mathsf{K}}(\mathbf{R}) \bigtriangledown_{\mathsf{R}}$$
$$Q = 2\phi_{\mathsf{K}}(\mathbf{R}) \bigtriangledown_{\mathsf{r}}.$$

In this paper we consider the capture of K⁻ mesons by the pairs of the nucleons in the 1p shell in nuclei such as ⁸Be, ¹²C, ¹⁴N and ¹⁶O. From the characters of Talmi coefficients, $2n_1+2n_2+l_1+l_2=2n+2N+l+L$ and $l_1+l_2=1+L=\lambda$, and spin-parity conservation, the quantum numbers of the pair coupled to the core are carefully selected. For the relative S-state pair these numbers are given in tables 1 and 2.

Table 1. The quantum numbers of the pair in the relative S state

| n | N | l | L | λ |
|---|---|---|---|---|
| 1 | 0 | 0 | 0 | 0 |
| 0 | 1 | 0 | 0 | 0 |
| 0 | 0 | 0 | 2 | 2 |

| Target nucleus | | $J_{ m pair}$ | λ |
|-------------------|--|-------------------|----------------|
| *Be J = 0+ | ${}^{6}\text{He} + (\text{pp})$ $J = 0^{+} s = 0$ ${}^{6}\text{Li} + (\text{pn})$ $J = 1^{+} s = 1$ | 0 + 1 + | 0 0, 2 |
| $J^{12}C = 0^+$ | $J^{10}Be + (pp)$ $J = 0^{+} s = 0$ $J^{10}B + (pn)$ $J = 3^{+} s = 1$ | 0 + 3 + | 0 2 |
| $J^{14}N = 1^{+}$ | $J^{12}B + (pp)$ $J = 1^+ s = 0$ $J^{12}C + (pn)$ $J = 0^+ s = 1$ | 0 + 2 + 1 + | 0 2 0, 2 |
| $J^{16}O = O^+$ | $J = 0^{+} s = 0$ $J = 0^{+} s = 0$ $J^{4}N + (pn)$ $J = 1^{+} s = 1$ | 0 + 1 + | 0 0, 2 |

Table 2. Spin and parity of target nuclei

Table 3. Number of pairs: Z and N denote the numbers of protons and neutronsin the 1p shell

| Pair | T_1 Λ_1 | sν | l | Weight | Number of pairs |
|------|-------------------|---|-------------|-----------------------------------|---------------------|
| pp | 1 1 | $egin{array}{ccc} 0 & 0 \ 1 & 0 \end{array}$ | even odd | $\frac{\frac{1}{2}}{\frac{1}{2}}$ | $\frac{1}{2}Z(Z-1)$ |
| | 1 0 | $\begin{array}{cc} 0 & 0 \\ 1 & \nu \end{array}$ | even odd | 14 34 | 1 \77 |
| pn | 0 0 | $\begin{array}{ccc} 1 & \nu \\ 0 & 0 \end{array}$ | even odd | 3 4 1 | 2112 |

In table 3 the numbers of the pairs existing in nuclei are given. Thus we obtain the transition rates for K^- meson absorption by pp and pn pairs:

$$W(\text{pp}; {}^{1}\text{S} \to {}^{3}\text{P}) = \delta_{L_{0}L_{0}}\delta_{M_{c}M_{0}}\delta_{S'S'} \frac{Z(Z-1)}{2} G^{2} \frac{16}{3} (2\pi)^{-4} \left(\frac{2\mu}{\hbar^{2}}\right)^{3/2} F$$
(8)

and

$$W(\text{pn}; {}^{3}\text{S} \to {}^{3}\text{P}) = \delta_{L_{c}L_{0}} \delta_{M_{c}M_{0}} \delta_{S'S'} \frac{NZ}{2} G^{2} \frac{32}{3} (2\pi)^{-4} \left(\frac{2\mu}{\hbar^{2}}\right)^{3/2} F$$
(9)

where

$$F = \int \left\{ m_{\rm K} c^2 - (M_{\Lambda} - M_{\rm p}) c^2 - B - E_{\rm x} - \frac{\hbar^2 K^2}{2(M_{\Lambda} + M_{\rm p})} - \frac{\hbar^2 K_{\circ}^2}{2M_{\circ}} \right\}^{1/2} \delta(\mathbf{K}_{\circ} + \mathbf{K})$$

$$\times \left| \int \exp(-i\mathbf{K} \cdot \mathbf{R}) \phi_{n'l'm'}(\mathbf{R}) \psi_{000}(\mathbf{R}) \, d\mathbf{R} \int j_0(kr) \frac{d}{dr} \varphi_{10}(r) r^2 \, dr \right.$$

$$- \int \exp(-i\mathbf{K} \cdot \mathbf{R}) \phi_{n'l'm'}(\mathbf{R}) \psi_{100}(\mathbf{R}) \, d\mathbf{R} \int j_0(kr) \frac{d}{dr} \varphi_{00}(r) r^2 \, dr \left|^2 d\mathbf{K} \, d\mathbf{K}_{\circ}. \quad (10)$$

Therefore the ratio between these transition rates is easily obtained:

$$\frac{W(\mathrm{pn}; {}^{3}\mathrm{S} \rightarrow {}^{3}\mathrm{P})}{W(\mathrm{pp}; {}^{1}\mathrm{S} \rightarrow {}^{3}\mathrm{P})} = \frac{2N}{Z-1}$$
(11)

where N and Z denote the numbers of the neutrons and protons in the 1p shell.

In the same way we can derive the ratios of the absorption rates by the nucleon pairs in the relative D state:

$$\frac{W(\text{pp; }^{1}\text{D} \rightarrow {}^{3}\text{F})}{W(\text{pp; }^{1}\text{D} \rightarrow {}^{3}\text{P})} = 1.5$$
(12)

and

$$\frac{W(\text{pn; }^{3}\text{D} \rightarrow {}^{3}\text{F})}{W(\text{pn; }^{3}\text{D} \rightarrow {}^{3}\text{P})} = 11.5.$$
(13)

3. Effects of the relative D-state pair on the capture rate

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In order to estimate the effects of the relative D-state pair, we must calculate the ratio of $W(\text{pn}; {}^{3}\text{D} \rightarrow {}^{3}\text{P})$ to $W(\text{pn}; {}^{3}\text{S} \rightarrow {}^{3}\text{P})$. For brevity, we consider the case of a Σ^{-} hyperon and a proton being emitted back to back after the K⁻ mesons are absorbed by ${}^{12}\text{C}$. We take the Gaussian-type function $f(r_{12}) = 1 - \exp(-\xi \mathbf{r}_{12})$ as the nuclear correlation function. In this case the capture rates are

$$\frac{dW(\mathrm{pn};\,{}^{3}\mathrm{S}\to{}^{3}\mathrm{P})}{d\cos\theta}\Big|_{\pi} = \frac{k^{6}}{75\times2^{5}}\frac{\mu}{\hbar^{2}}\frac{G^{2}}{6}\frac{\pi}{(2\pi)^{3}}A^{2}N_{\mathrm{S}}^{2}$$
$$\times \left|\gamma^{-3/2}\exp\left(-\frac{k^{2}}{4\gamma}\right) - \eta^{-3/2}\exp\left(-\frac{k^{2}}{4\eta}\right)\right|^{2} \qquad (14)$$

and

$$\frac{dW(\mathrm{pn; }^{3}\mathrm{D} \rightarrow {}^{3}\mathrm{P})}{d\cos\theta}\Big|_{\pi} = \frac{k^{8}}{5\times2^{9}}\frac{\mu}{\hbar^{2}}\frac{G^{2}}{G^{2}}\frac{\pi}{(2\pi)^{3}}B^{2}N_{\mathrm{D}}^{2}$$
$$\times \left|\gamma^{-7/2}\exp\left(-\frac{k^{2}}{4\gamma}\right) - \gamma^{-7/2}\exp\left(-\frac{k^{2}}{4\eta}\right)\right|^{2}$$
(15)

where

$$A = \int \phi_{n'l'm'}(\mathbf{R})\psi_{02M}(\mathbf{R}) d\mathbf{R}, \qquad B = \int \phi_{n'l'm'}(\mathbf{R})\psi_{000}(\mathbf{R}) d\mathbf{R}$$

$$N_{\rm S}^{-2} = \frac{1}{4} \left\{ \frac{\pi}{(2\gamma)^3} \right\}^{1/2} \left\{ 1 - 2 \left(\frac{2\gamma}{2\gamma + \xi} \right)^{3/2} + \left(\frac{\gamma}{\gamma + \xi} \right)^{3/2} \right\}$$

$$N_{\rm D}^{-2} = \frac{15}{16} \left\{ \frac{\pi}{(2\gamma)^7} \right\}^{1/2} \left\{ 1 - 2 \left(\frac{2\gamma}{2\gamma + \xi} \right)^{7/2} + \left(\frac{\gamma}{\gamma + \xi} \right)^{7/2} \right\}$$

$$\epsilon = m_{\rm K} c^2 - (M_{\Lambda} - M_{\rm p}) c^2 - B - E_{\rm X}$$

$$k^2 = 2\mu\epsilon/\hbar^2$$

$$G = \frac{\hbar^2 g_{\Sigma\rm NK}}{2M_{\rm p} c (2m_{\rm K})^{1/2}} \frac{M_{\Sigma} - M_{\rm p}}{M_{\Sigma}} \qquad \left(\frac{g_{\Sigma\rm NK}^2}{4\pi\hbar c} = 2 \right)$$
(16)

and θ is the opening angle between two particles emitted from the nucleus. It is assumed that each nucleon moves in the harmonic oscillator potential characterized by the spring constant α . γ is related to α , i.e. $\gamma = \frac{1}{2}\alpha$. In (16) the wave functions of the centre-of-mass motion of the pair are taken to be of the harmonic oscillator type with the constant $\beta = 2\alpha$.

Using the value $\alpha = 0.187 \text{ fm}^{-2}$ determined by the electron scattering experiments for ¹²C (Hofstadter 1957), we obtain the ratio of the capture rate by the relative D-state pair to that by the relative S-state pair,

$$R = \frac{W(\text{pn}; {}^{3}\text{D} \rightarrow {}^{3}\text{P})}{W(\text{pn}; {}^{3}\text{S} \rightarrow {}^{3}\text{P})}$$

which is equal to 2.90×10^{-4} , 0.879×10^{-3} and 1.66×10^{-2} for n' = 3, 4 and 5, respectively, without the nuclear correlation, i.e. $\xi = \infty$. This value is very small. K mesons are captured in a much higher orbit than that of π mesons (Baker 1960, Martin 1963). Since the momentum transfer to the nucleons in the processes under consideration is large, only highly correlated pairs of nucleons with separation of the order 0.4 fm seem to exist. Recently it was shown that the nuclear correlation is important in the π^- -meson absorption (Cheon 1966). This ratio must be very sensitive to the short-range nuclear correlations. With the nuclear correlation included, we obtain $R = 1.93 \times 10^{-8}$, 0.585×10^{-7} and 1.10×10^{-6} for n' = 3, 4 and 5, respectively, for $\xi(^{3}S) = \xi(^{3}D) = 1.8$ fm², which was determined from the stopped π^- -meson absorption (Cheon 1967). Therefore we can easily obtain the ratio of K⁻ capture by the relative D-state pair to the relative S-state pair,

$$R' = \frac{W(\text{pn; }^{3}\text{D})}{W(\text{pn; }^{3}\text{S})} = \frac{W(\text{pn; }^{3}\text{D} \to {}^{3}\text{P}) + W(\text{pn; }^{3}\text{D} \to {}^{3}\text{F})}{W(\text{pn; }^{3}\text{S} \to {}^{3}\text{P})}$$

which is equal to 3.63×10^{-3} , 1.10×10^{-2} and 0.208 for n' = 3, 4 and 5, respectively, without the effects of nuclear correlation being included. If the nuclear correlation is included, $R' = 2.41 \times 10^{-7}$, 0.731×10^{-6} and 1.38×10^{-5} for n' = 3, 4 and 5, respectively.

4. Calculation of the absorption rate in the process $K^- + {}^{12}C \rightarrow {}^{10}B + \Sigma^- + p$

Since only the charged particles are emitted, measurements are easier for process (6) than for other processes. In this section we calculate the absolute value of the absorption rate. The interaction of the process is

$$H^{\Sigma \mathrm{NK}} = i g_{\Sigma \mathrm{NK}} \bar{\psi}_{\Sigma} \gamma_5 \phi_{\mathrm{K}} \psi_{\mathrm{n}}.$$

The initial and final states are given by (3) and (4). Since K⁻ mesons are absorbed by the (pn) pair, the quantum numbers of the pair can be found from tables 1 and 2, i.e. n = N = 0, l = 0, L = 2 and $\lambda = 2$. We make the approximation that K⁻ mesons are absorbed only by the relative S-state pair. This approximation is valid from the discussion in the previous section. The absorption rates are given in tables 4 and 5 for the case of the sigma and proton being emitted back to back leaving the residual nucleus. Table 4 gives the results without the nuclear correlations being taken into account, and table 5 presents those with the nuclear correlations included, $\xi({}^{3}S) = \xi({}^{3}D) = 1.8 \text{ fm}^{-2}$.

Table 4. The absorption rates in the process $K^- + {}^{12}C \rightarrow {}^{10}B + \Sigma^- + p$ in units of s⁻¹: nuclear correlations are not considered

| n' | 3 | 4 | 5 |
|---|--------------------------|-------------------------|------------------------|
| $\frac{dW({}^{3}S \rightarrow {}^{3}P)}{d\cos \theta}\Big _{\theta = \pi}$ | 2.886×10^{13} | 5.517×10^{12} | 2.325×10^{12} |
| $\frac{dW(^{3}S \rightarrow {}^{1}S)}{d\cos\theta}\Big _{\theta=\pi}$ | 4.6777 ×10 ¹³ | 3·410 ×10 ¹³ | 1.144×10^{14} |
| $\frac{dW({}^{3}\mathrm{D} \rightarrow {}^{3}\mathrm{P})}{d\cos\theta}\Big _{\theta=\pi}$ | 0.8363×10^{10} | 0.4852×10^{10} | 3.859 × 1010 |
| Total | 0.7564×10^{14} | 0.3962×10^{14} | 1.167×10^{14} |

Table 5. The absorption rates in the process $K^- + {}^{12}C \rightarrow {}^{10}B + \Sigma^- + p$ in units of s^{-1} : nuclear correlations are considered

| 'n | 3 | 4 | 5 |
|---|--------------------------|-------------------------|--------------------------|
| $\frac{dW({}^{3}S \rightarrow {}^{3}P)}{d\cos\theta}\Big _{\theta=\pi}$ | 5·206 × 10 ¹⁷ | 0.9952×10^{17} | 0.4194 ×10 ¹⁷ |
| $\frac{dW(^{3}S \rightarrow {}^{1}S)}{d\cos\theta}\Big _{\theta=\pi}$ | 0.8432×10^{18} | 0.6511 ×1018 | 2.063 × 1018 |
| $\frac{dW({}^{3}\mathrm{D} \rightarrow {}^{3}\mathrm{P})}{d\cos\theta}\Big _{\theta=\pi}$ | 1.004×10^{10} | 0.5825×10^{10} | 4.633 ×1010 |
| Total | 1.364×10^{18} | 0.7146 × 1018 | 2.105×10^{18} |

5. Concluding remarks

In §2 we calculated the ratio $R_0 = W(pn \to \Lambda n)/W(pp \to \Lambda p)$. However, it is rather difficult to distinguish the process $K^- + 2N \to \Lambda + N$ from the process $K^- + 2N \to \Sigma^0 + N$ experimentally. Fortunately, since the ratio R depends only on the spin of particles,

$$R_{0} = \frac{W(\mathrm{pn} \to \Lambda \mathrm{n})}{W(\mathrm{pp} \to \Lambda \mathrm{p})} = \frac{W(\mathrm{pn} \to \Sigma^{0} \mathrm{n})}{W(\mathrm{pp} \to \Sigma^{0} \mathrm{p})}$$

Therefore

$$\frac{W(\mathrm{pn} \to \Lambda \mathrm{n}) + W(\mathrm{pn} \to \Sigma^{0} \mathrm{n})}{W(\mathrm{pp} \to \Lambda \mathrm{p}) + W(\mathrm{pp} \to \Sigma^{0} \mathrm{p})} = \frac{R_{0}W(\mathrm{pp} \to \Lambda \mathrm{p}) + R_{0}W(\mathrm{pp} \to \Sigma^{0} \mathrm{p})}{W(\mathrm{pp} \to \Lambda \mathrm{p}) + W(\mathrm{pp} \to \Sigma^{0} \mathrm{p})} = R_{0}$$

Thus we can obtain R experimentally without distinguishing the process $K^- + 2N \rightarrow \Lambda + N$ from the process $K^- + 2N \rightarrow \Sigma^0 + N$. European K^- -collaboration (1959) investigated the emission of Σ hyperons from K^- capture at rest in a nuclear emulsion. It was shown by them that about 16% of identified charged Σ hyperons were attributed to processes of no π production. Furthermore, they obtained the following results.

When the total numbers of two-nucleon processes leading to Σ -hyperon production (Σ^{\pm} and Σ^{0}) in the stars are 300, the numbers of the final state $\Sigma^{+} + n$ are 50 and the numbers of the final state $\Sigma^{-} + p$ are 60. Then we obtain the ratio between the K⁻-meson absorption rates:

$$\frac{W(\mathrm{K}^- + \mathrm{p} + \mathrm{n} \rightarrow \Sigma^- + \mathrm{p})}{W(\mathrm{K}^- + \mathrm{p} + \mathrm{p} \rightarrow \Sigma^+ + \mathrm{n})} = \frac{6}{5}.$$

Assuming the charge independence of the processes, we may compare our theoretical result with this experimental result. $R^{\text{theor}} = 2.7$, 2.5 and 2.4 for ¹²C, ¹⁴N and ¹⁶O, respectively. The theoretical values are about twice as large as the experimental values.

Common and Higgins (1964) calculated the non-mesonic absorption rates by ⁴He. The total absorption rates obtained by them using a Gaussian wave function are

$$W(K^- + {}^{4}He \to \Sigma^- + p + (n+p)) = 6 \cdot 01 \times 10^{17} \, s^{-1}$$
$$W(K^- + {}^{4}He \to \Sigma^- + n + (p+p)) = 0 \cdot 89 \times 10^{17} \, s^{-1}.$$

These values are based on the assumption that the capture of K⁻ mesons in helium is predominantly from S orbitals. The experimental values of the absorption rate from the 1S level are smaller than $2 \times 10^{18} \text{ s}^{-1}$. Our results are tabulated in the table 4. The absorption rates are very sensitive to the nucleon correlation. We have used the values of correlation parameters determined from the stopped π^- -meson absorption (Cheon 1967). Our results seem to be reasonable compared with the results of Common and Higgins.

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